Dielectric Response of a Planar Periodic Array of Polarizable Wires Parallel to an Interface with a Nonlocal Dynamic Plasma-like Medium

Abstract—We examine the spatially inhomogeneous polarizability and dielectric function of the combined Coulomb-coupled system of a periodic lateral lattice array of polarizable 1D wire plasmas and a semi-infinite bulk plasma nearby. Furthermore, we have solved the Random Phase Approximation (RPA) integral equation for the inverse dielectric screening function, \( K(y_1, z_1; y_2, z_2; q, \omega) \), of the combined system assuming no overlap of the wave functions of the wire lattice with those of the semi-infinite plasma (no tunneling or charge exchange). The solution is given by

\[
K(y_1, z_1; y_2, z_2; q, \omega) = K_{\text{semi}}(y_1, z_1; y_2, z_2; q, \omega) \sum_{n=-\infty}^{\infty} C(y_1 - na; z_1) \frac{a}{2\pi} \int_{-\pi/a}^{\pi/a} dp e^{-ipna} \frac{\tilde{K}_{\text{semi}}(p, z_0; y_2, z_2)}{1 - 2e^{2} \sum_{r=-\infty}^{\infty} e^{ipra} e^{ipra} C(ra; z_0)},
\]

where \( K_{\text{semi}}(y_1, z_1; y_2, z_2; q, \omega) \) is the corresponding screening function of the semi-infinite solid state plasma in the absence of the quantum wire lattice and

\[
C(y_1 - na, z) = \frac{n_1 \rho_q^2}{\omega_0^2} \int dz' \int dY \times K_{\text{semi}}(y_1 - na - Y; z, z') \times K_0(|q| \sqrt{Y^2 + (z' - z_0)^2}),
\]

where \( K_0 \) is the MacDonald function of order zero. In Eq.(2),

\[
\tilde{K}_{\text{semi}}(p, z_0; y_2, z_2) = \sum_{r=-\infty}^{\infty} e^{ipra} K_{\text{semi}}(ra, z_0; y_2, z_2),
\]

in consequence of periodicity of the quantum wire superlattice. Details of the derivation of this result will be presented elsewhere[1].

I. INTRODUCTION

The dynamic, nonlocal and spatially inhomogeneous dielectric response of a planar, lateral, periodic lattice of polarizable wires outside a semi-infinite plasma-like medium is analyzed here. The individual quantum wires of the lattice are taken to embody one-dimensional (1D) plasmas which depend on frequency and 1D wavenumber, and the plane of the array of wires is parallel to the bounding surface of the nearby semi-infinite plasma-like medium containing Coulomb-coupled interacting electrons constituting a semi-infinite dynamic, nonlocal bulk plasma. Taking the wires in the x-direction and the array of wires in the x-y plane at distance \( z_0 < 0 \) from the bounding surface of the semi-infinite semiconductor plasma (the wires are situated at \( y = na \), with \( n = \ldots, -1, 0, 1, \ldots \)), we Fourier transform in the x-direction of complete translational symmetry \( x \rightarrow x' \) and in time \( (t - t' \rightarrow \omega) \) to determine the solution for the screening function, \( K(y_1, z_1; y_2, z_2; q, \omega) \), as the space-time matrix inverse of the direct dielectric function \( \varepsilon(\vec{r}, t; \vec{r}', t') \),

\[
\int d^3xd\tau K(\vec{r}, t; \vec{r}', \tau)\varepsilon(\vec{r}, \tau; \vec{r}', t') = \delta(\vec{r} - \vec{r}')\delta(t - t') \tag{1}
\]
(we take the region \( z < 0 \) to be vacuum except for the wire lattice structure.)

\[
1 = \frac{2e^2 n_1 D q_z^2}{m \omega^2} \left[ \sum_{r=-\infty}^{\infty} e^{ir \cdot |a|} K_0(|r|) \right] + \frac{\pi}{a} \Gamma \sum_{n=-\infty}^{\infty} \frac{e^{-2|n| \sqrt{(p-2n\pi/a)^2 + q_z^2}}}{\sqrt{(p-2n\pi/a)^2 + q_z^2}},
\]

(5)

where \( \Gamma \), the semi-infinite plasma image strength function, is given by

\[
\Gamma = \frac{\omega_p^2}{2 \omega^2 - \omega_p^2},
\]

(6)

with \( \omega_p \) as the classical plasma frequency of the semi-infinite bulk taken in the local limit for illustrative purposes. As Eq. (5) is quadratic in \( \omega^2 \), it can be solved exactly with the result (subject to \( \omega_p^2 > 0 \)),

\[
\omega_\pm^2 = \frac{1}{2} \left[ AB + \frac{\omega_p^2}{2} \pm \sqrt{A^2 B^2 + 2AC - AB \omega_p^2 + \frac{\omega_p^4}{4}} \right],
\]

(7)

where

\[
A = \frac{2e^2 n_1 D q_z^2}{m},
\]

(8)

\[
B = \sum_{r=-\infty}^{\infty} e^{ir \cdot |a|} K_0(|r|)
\]

(9)

\[
C = \frac{\pi}{a} \omega_p^2 \sum_{n=-\infty}^{\infty} \frac{e^{-2|n| \sqrt{(p-2n\pi/a)^2 + q_z^2}}}{\sqrt{(p-2n\pi/a)^2 + q_z^2}}
\]

(10)

Noting that the series involved in the solution for \( \omega_\pm^2 \) are periodic in the reciprocal lattice wavenumber \( p \to p - \frac{2\pi}{a} \), we obtain two bands of plasmon modes in the first Brillouin zone, \( -\pi/a \leq p \leq \pi/a \). The gap separating the two bands may be described in terms of

\[
\Delta(\omega^2) \equiv \omega_+^2 - \omega_-^2 = \sqrt{A^2 B^2 + 2AC - AB \omega_p^2 + \frac{\omega_p^4}{4}}.
\]

(11)

In connection with Eq. (9), it should be noted that the \( r = 0 \) term on the right hand side indicates an apparent logarithmic divergence of \( K_0(0) \). However, this divergence is an artifact of considering the quantum wire to have zero width. Adjusting this to a finite width, \( b \), we have the finite result \( K_0(q_z b) \), and

\[
B = K_0(q_z b) + 2 \sum_{r=1}^{\infty} \cos(rp) K_0(rq_z b).
\]

(12)

III. CONCLUSIONS

Based on Eqns. (7-12), we have calculated the coupled plasmon roots, \( \omega_\pm/\omega_s \), normalized to the surface plasmon frequency

\[
\omega_s = \omega_p/\sqrt{2}.
\]

(13)

Figure 1 exhibits the two plasmon bands and the gap between them in the first Brillouin zone, \( -\pi/a \leq p \leq \pi/a \), using the parameters \( n_1 D = 10^8/m; m = .067 m_e; q_z = 0.1q_F; q_F = 1.7 \times 10^8/m \), period of the wire lattice is \( a = 10nm \); and separation of the lattice from the surface is given by \( z_0 = -1nm \).

It is of particular interest to note the width of the gap reflects upon the lattice period, \( a \), because of its variation with \( a \). This is illustrated in Figure 2 using the same parameters, except \( p = 2 \times 10^8/m \), over the range of \( a \) from (almost) 0 to 2 \( \times 10^{-7}m \). The strongest variation occurs for lattice spacings below 100nm. The hybridization of the surface plasmon and wire-lattice plasmon occurs principally for separations below about \( |z_0| \leq 50nm \), and they effectively decouple for larger separations for the parameters under consideration.

These considerations exemplify the fact that an adsorbed layer/lattice shift the surface plasmon and mix it with the oscillatory modes of the adsorbate, introducing new coupled modes. In essence, the adsorbate modifies all dielectric response properties of the system and, consequently, changes its optical properties (for example, in surface plasmon resonance experiments) providing information about the adsorbate itself.

The determination of the screening function, \( K_0 \), (as in Eq. (2) in the case at hand) not only provides the shifted collective mode spectrum in terms of its frequency poles, but it also provides the relative oscillator strengths of the various modes.
in terms of the residues at the poles.

The analytic technique employed here for a periodic lateral quantum wire lattice can be extended to finite numbers of wires, thin layers, dot-like structures, as well as a few localized molecular oscillators. Furthermore, it is readily adjusted to take account of spatial nonlocality, phonons, etc. This phenomenology can also be tuned by the introduction of a magnetic field.

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